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Nutational Flows Inside Spinning Cylinders

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NOTICE

FOREWORD

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A review was conducted for the coning anomaly of the PAM vehicles. The focus was on the nutational flows inside spinning cylinders, fully or partially filled, with or without exit holes. The surveyed results suggested a new hypothesis for the PAM coning. In this hypothesis, a rather abrupt transition from a forced vortex motion to a free vortex motion occurs inside the spinning solid rocket motor at Ro = 0.6-0.8. The vortex after transition supports helical waves. As the helical waves travel along the vortex, they precess in the retrograde direction at a frequency close to the PAM coning frequency. This leads to a resonant interaction with the PAM vehicle and the subsequent coning growth.

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NOMENCLATURE

d	nozzle diameter
m	plate oscillation frequency
n	grid oscillation frequency
p	pressure perturbation
u	rms turbulent velocity
Cp	pressure coefficient, $p/\theta\rho\Omega^2R^2$
D	cylinder diameter
R	cylinder radius
Re	Reynolds number, $\Omega R^2/\nu$
Ro	Rossby number based on axial velocity, $U/\Omega R$
Ro_b	Rossby number based on surface burning velocity, $U_b/\Omega R$
Ro_g	grid Rossby number, $n/2\Omega$
Ro_ℓ	local Rossby number, $u/\Omega\ell$
U	axial velocity
U_b	surface burning velocity
ℓ	integral length scale of turbulence
ω	precession frequency
θ	half cone angle
ν	kinematic viscosity
ρ	density
Ω	cylinder spin rate

Subscripts:

b	burning surface
g	grid
l	local
p	pressure

Nutational Flows Inside Spinning Cylinders

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ABSTRACT

A review was conducted for the coning anomaly of the PAM vehicles. The focus was on the nutational flows inside spinning cylinders, fully or partially filled, with or without exit holes. The surveyed results suggested a new hypothesis for the PAM coning. In this hypothesis, a rather abrupt transition from a forced vortex motion to a free vortex motion occurs inside the spinning solid rocket motor at Ro ≈ 0.6-0.8. The vortex after transition supports helical waves. As the helical waves travel along the vortex, they precess in the retrograde direction at a frequency close to the PAM coning frequency. This leads to a resonant interaction with the PAM vehicle and the subsequent coning growth.

1. Introduction

For decades, spin has been employed to solid rocket motors to provide dynamic stability or to reduce the effect of thrust misalignment.[1] Depending upon the spin rate, propellant formulation, and motor configuration, the dynamic performance of a solid rocket motor can exhibit gross deviations from its static performance. Many anomalies have been

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experienced, especially after the use of aluminum and other additives to achieve higher burning temperatures. Among them is a rapid coning growth near the end of motor burn of the PAM-D and PAM-DII spacecraft.[2-10] This coning phenomenon is illustrated in Figure 1. It is a nutation, or precession, of the spin axis of the spacecraft around the flight direction; it is like a wobbling football. The coning growth rate has a sharp rise near the end of the motor burn, as in resonance, as seen in the sample time history in Figure 2. The final half cone angle can be as high as 20° in some missions, and could have endangered the missions. Such a problem was not noticeable in earlier systems utilizing smaller but otherwise similar solid rocket motors; it is a problem of large-size motors. However, because of the increasing payloads, the large motors will continuously be used in future missions, thereby demanding a solution to the coning anomaly.

So far, most flight data regarding the coning anomaly were obtained by the rate gyroscope or the accelerometer fixed on the spacecraft. They consistently indicated the correlation between the coning growth and some flow instability inside the motor. The flow instability could be the sloshing of the liquid aluminum oxide which accumulates during the motor burn.[11-15] The slag sloshing generates an offset in the mass center of the spacecraft and results in a thrust-induced lateral torque, as shown in Figure 3(a). The instability could also be a wave motion of the internal gas flow. In Flandro's jet gain model,[4, 7-9] a vorticity wave nutating in the combustion chamber, especially in the vicinity of the submerged nozzle entrance, is responsible for the disturbing torque; see Figure 3(b). Or[16-17] on the other hand suggested that the disturbing torque comes from a jet deflection presumably related to flow separation, as shown in Figure 3(c). These models, some having been studied for years, still

lack of conclusive experimental verification. A re-examination of these models and, more fundamentally, the flows inside spinning cylinders is thus necessary.

The review will start with the flows inside spinning cylinders with no exit hole, either fully or partially filled, which have been the focus of most theoretical analyses. It then examines the flows inside spinning cylinders with an exit hole, to which the spinning rocket motors bear more resemblance. The discussions will be confined to the right circular cylinders, unless specified otherwise. Whenever suitable, the cylinder flows will be compared with the flows inside the laboratory models of solid rocket motors or the real solid rocket motors.

2. Flows in spinning cylinders with no exit hole

Coning has been a problem not only for the PAM vehicle but also for spinning shells and projectiles with liquid payloads. This has led to a significant number of coning analyses and experiments on the closed, spinning cylinder, many performed by the group at the Army Ballistic Research Laboratory. Depending upon whether the spinning container is fully or partially filled, coning could result from sloshing or nutational inertial waves inside the spinning cylinder.

2.1 Fully-filled cylinders

Figure 4 shows the flow during spin up inside a closed, fully-filled cylinder.[18] As seen in the figure, boundary layers form on the horizontal surfaces inside the cylinder, and the nonrotating fluid in the core region flows toward the boundary layers and returns to the interior via the vertical side wall, with spinning. The process continues until all of the fluid inside the spinning cylinder is in solid body rotation.

The solid-body rotation in a fully filled cylinder supports inertial waves.[18-65] One classical example is the wave motion generated by a horizontal disk oscillating at a frequency m in the direction parallel to the rotation axis,[30-31] as shown in Figure 5. As seen in the Figure, under excitation, the internal waves appear in the form of axisymmetric cones when $m \leq 2\Omega$, and disappear when $m > 2\Omega$ (Ω, the cylinder rotation speed). Clearly, the wave amplification depends on the cylinder rotation. So does the coning instability except that the waves involved are the nutational inertial waves, which, as will be seen, can be responsible for the coning growth during the spin up and the solid body rotation.

2.1.1 Nutational waves in solid body rotation

It is well known that the solid body rotation inside a spinning cylinder supports nutational inertial wave motions forced either by tilting the cylinder or by slanting part of its boundary.[66-123] These wave motions have been studied for liquid payloads inside spinning shells or projectiles. The analyses fall into two categories. One assumes the importance of viscous force everywhere in the flow. The other assumes that the viscous force is important only in boundary layers and shear layers. Most analyses belong to the second category, the backbone of which is the balance between the pressure gradient force and the Coriolis force.

Both the eigenvalue computations and the time-dependent numerical analyses have consistently indicated that the coning growth of the liquid-filled, rotating cylinder is caused mainly by the resonance between the natural modes of nutational waves of the solid body rotation and the external coning excitations.[72, 82, 98, 105] The resonance was verified experimentally by measuring the pressure coefficient Cp ($\equiv p/\theta\rho\Omega^2R^2$; p, the pressure perturbation amplitude; θ, the half cone angle; ρ, the density; Ω, the cylinder rotation speed;

and R, the cylinder radius) inside a gyroscope[104] (Figure 6). Figure 7 shows one typical result obtained in the experiment. As seen in the figure, Cp reaches the maximum when the coning frequency ω of the gyroscope is equal to the eigenfrequency of the spinning liquid predicted by the theory. The resonance depends upon the Reynolds number Re ($= \Omega R^2/\nu$; R, the cylinder radius; and ν, the kinematic viscosity).[121] When Re > 1000, the wave motion can be well predicted by the resonance theory. However, when Re < 1000, the growth rate of coning angle was found to increase monotonically with the coning frequency.

The eigenvalues and the roll and side moments of the nutational disturbances have been calculated with increasingly sophisticated means. Stewartson[72] started the computation for an inviscid payload. Then, Wedemeyer[84] made the viscous corrections, and Murphy[118] included all pressure and wall shear contributions. The Stewartson-Wedemeyer eigenvalue calculation was later improved by replacing the cylindrical wall boundary approximation with a linearized Navier-Stokes approach,[99] followed by time-dependent numerical analyses[113, 120] and three-dimensional Navier-Stokes simulation.[123]

For the highly viscous liquid, instead of the resonance mechanism, Herbert[124-127] proposed an average internal circulation as the cause of the coning growth and despin moment. Without considering the instabilities, the calculated despin moment appears to agree well with data obtained by experiments[128-133] and numerical analyses[134-135] over a wide range of Reynolds number.

The investigation of coning growth was extended to the spin up stage of the rotating cylinder with liquid payload.[136-161] D'Amico[151] found in his pressure measurement that the cone-up time of the liquid in a spinning cylinder is comparable to its spin-up time, thus

suggesting a resonance between the nutational waves and the external coning excitations. This is consistent with the outcome of theoretical and numerical analyses.[153-154] Note that during the early spin up stage, the critical layer forms in the rotating flow. The eigenfrequency and moments of the spinning liquid have thus been computed separately for the early spin up stage when there is a critical layer[159] and the later stage when the critical layer no longer exists.[160]

2. 2. 2 Nutational waves on vortices

The nutational waves discussed so far are supported by the solid body rotation or the spin-up of the fluid; that is, by the primary motion inside the spinning cylinder. A special class of nutational waves however are supported by vortices formed via the secondary flow in the rotating cylinder.[162-175]

For an axisymmetric rotating flow, Scorer[166] had speculated that the small-scale turbulence has the effect to redistribute the vorticity to form vorticity concentrations at the center and boundaries. This effect was studied experimentally.[167-169] It was McEwan[169] who first showed local vorticity concentrations in the spinning cylinder, the intensity of which was 2-3 times the background vorticity 2Ω. Much stronger vorticity concentrations were later observed by Hopfinger, Browand and Gagne[171] (referred as HBG hereafter). In their experiment, the turbulence was produced with an oscillating grid at the bottom of a deep, rotating water tank. Near the grid, the Rossby number Ro_g ($\equiv n/2\Omega$; n, the grid oscillation frequency) was kept large such that the turbulence was locally unaffected by rotation. Away from the grid, the turbulence intensity decreased and the rotation became important. As the local Rossby number Ro_ℓ ($\equiv u/\Omega\ell$, where u is the rms turbulent velocity and ℓ the integral

scale of turbulence) decreased to about 0.4, a rather abrupt transition occurred. The flow after transition consisted of concentrated vortices, having axes approximately parallel to the rotation axis and extending through the fluid above the turbulent Ekman layer. Figure 8 shows the cross-sectional views of the flows with and without the tank rotation. As seen in the Figure, the vortices form only in the rotating flow. The observed vo-·'city concentrations were about 50 times the tank vorticity 2Ω. As speculated, those vorticity concentrations are formed by the local vorticity convention induced either by the propagation of the finger-like turbulence front,[173] shown schematically in Figure 9, or by the grid suction effect.[174]

The observed vorticity concentrations support nutational waves. Figure 10 shows a spiral wave travelling along a vortex. Such spiral waves actually nutate as they travel along the vortices. This is shown schematically in Figure 11, which also includes other spiral configurations observed in the same experiment. Since in this experiment waves of opposite spiral configurations and nutation directions occur simultaneously over the cross-section of the rotating tank, no net coning effect is expected. However, this is not the case for a similar phenomenon in the spinning cylinder having an exit hole, as will be seen in Section 3.

2.2 Partially-filled cylinders

Besides the fully filled cylinder, the partially filled, spinning cylinder also experiences coning growth. The liquid sloshing has long been suspected as the cause, and substantial amount of literature on sloshing is available.[176-217]

Comprehensive reviews on liquid sloshing have been reported before.[187, 193] Low-frequency sloshing modes were observed in containers with asymmetric boundaries,[192, 194] or by coning the cylinders. Figure 12 shows a sloshing motion visualized in a circular cylinder. As nutational waves in the fully filled cylinder, the liquid sloshing can be much amplified by the resonance between the natural sloshing frequency of the spinning liquid and the external coning frequency.[11-14, 212, 215] The resonance causes a significant offset of the mass center of the liquid inside the spinning cylinder and the nutation of the spinning cylinder. This mechanism is believed to be responsible for the conings of the partially filled spinning shells and satellites. Similar sloshing of the aluminum slag in the solid rocket motor may also cause the PAM coning, although its consistency with the flight test data is still in doubt.

In analyzing the sloshing motion, usually, a simple pendulum model consisting of springs and dashpots is appropriate.[13] However, this representation may not appeal in complicated situations, for instance, in a spin-stabilized satellite comprising off-axis tanks.[203] It may need corrections for the dependence of the resonance modes on the contact line position[214, 216] or the dependence of the frequencies on the container flexibility.[198]

Forces and moments due to liquid sloshing have been computed using sophisticated means[189-190, 200] and formulas have been compiled. The computations have been extended to the spin-up cases,[213] and to complex cases like the unsteady incompressible flow[217] and the axisymmetric three-dimensional transient flow,[214] using numerical simulation.

3. Flows in spinning cylinders with an exit hole

For the spinning rocket motor, the spinning cylinder with an exit hole is a more realistic model for analysis than its closed counterpart. Most analyses have been done on

the internal gas flows. Noticeable among them are Flandro's jet gain model[4, 7-9] and Or's jet deflection model.[16-17] They are examined below, followed by a collection of experimental results to form a new hypothesis for the PAM coning.

3.1 Gas dynamics analyses

The central idea in Flandro's jet gain model is that the pressure distribution inside a spinning rocket motor, especially in the vicinity of the submerged nozzle entrance, is modified by the Coriolis force acting on the gas traversing through the wobbling chamber and hence results in destabilizing moments on the spacecraft.[9] The analysis started with the classical jet damping mechanism. As concluded in the analysis, if the gas flow is steady and uniform with respect to the wobbling chamber, the Coriolis acceleration due to wobbling is balanced by a wave-like pressure disturbance (see Figure 13). The integrated effect of the pressure distribution results in a torque on the chamber which is opposite to the chamber lateral angular velocity and hence stabilizes the nutation, as shown in Figure 14. That is, the jet damping is simply the reaction torque to resist the wobbling.

As the chamber size increases beyond a critical size, the internal gas flow becomes increasingly sensitive to the vehicle motion, and is unsteady and three-dimensional. The Coriolis acceleration due to wobbling is thus balanced by both the pressure and velocity perturbations as the momentum equations require. The wobbling in this case induces unsymmetrical vorticity waves, which precess in the retrograde direction about the chamber axis, as shown in Figure 15. The associated pressure waves result in a torque driving the

nutation and destabilizing the chamber when the waves are in resonant coincidence with the vehicle precession frequency.

In the analysis, the modes of wave motions are determined by the governing equations for the natural modes inside a spinning cylinder with no exit hole. The approach is basically the same as the analyses for liquids in spinning shells or projectiles reported in Section 2.1. This leads to the Poincaré wave equation, the solutions of which have frequencies in the same range as the spacecraft nutation frequency. The mean flow is then included as the forcing term in the wave equation. Three mean flows are considered. They are the solid body rotation, the free vortex, and the free vortex with radially inward motion. From the analysis, two resonant interactions are predicted for the PAM-D vehicle and one of them is close to the flight data, as shown in Figure 16.

Exact solutions to the Poincare wave equation can be found only in simple geometries. A full-scale three-dimensional Navier-Stokes numerical algorithm has thus been developed. In addition, both the cold-flow and hot-flow experiments were conducted to test the analytical model.

Besides the jet gain model, Or[16-17] has developed a different gas dynamics model which suggests a jet deflection inside the motor, presumably related to the flow separation, as the mechanism for the coning growth. This model, shown schematically in Figure 4(c) before, is simple in analysis but provides no leads to the wavy behavior of the jet deflection and the characteristic frequency as in Flandro's model, which nevertheless are necessary for comparison with the the flight test data.

The two gas dynamics models examined thus far need experimental verifications. Meanwhile, the flight data of the PAM vehicles appear to be consistent with a collection of laboratory results.

3.2 Transition from forced vortex to free vortex

In early 70's, Dunlap[218] investigated the swirling flow in the spinning end-burning solid rocket motor. The experiment was conducted in a cylindrical cold-flow model with a porous plate at one end to simulate the end burning. Both smoke visualization and pressure measurements were conducted. Figure 17 shows the smoke patterns observed near the end plate at increasing spinning rates. As seen in the figure, at low spin rates, the flow is in solid body rotation in unison with the rotating model. As the model spin rate increases to 1,000 rpm, the smoke plume begins to bend radially inward and in the direction of the rotation. And at 1,500 and 2,000 rpm, stagnant smoke is seen near the chamber walls, indicating flow separation there. However, by moving the smoke plume closer to the center at the spin rate of 2,000 rpm, again the lower part of the smoke near the plate is seen to accelerate radially inward and spiral up the center, whereas the outer part moves mostly in the spin direction, as shown in Figure 18. The spiral vortex flow was modelled by Dunlap, as shown in Figure 19, in accordance with the smoke visualizations. Based on the pressure measurements at the centerline he further concluded that the onset of transition to the spiral vortex flow in the chamber occurred at a Rossby number $Ro \approx 0.52$ ($Ro \equiv U/\Omega R$; U, the axial velocity; and R, the cylinder radius), and the transition was unaffected by the nozzle geometry, chamber length, and a two-fold increase in Reynolds number.

Complementary to this study were the measurements by Johnson and L'Ecuyer[219] of the axial and tangential velocities in a cold-flow solid rocket model similar to the one used by Dunlap. The velocities were measured by a five-port impact tube at a station downstream of the end porous plate for two nozzle contraction ratios, 5.25:1 and 22.1:1. The results are shown in Figure 20. As seen in the figure, at the contraction ratio 5.25, the tangential velocity increases linearly with the radius as in the forced vortex motion (solid body rotation) for all three rotation speeds tested, and the axial velocity is nearly uniform across the chamber. However, as the contraction ratio raises to 22.1:1 ratio, both the axial and tangential velocities behave like a free vortex motion at higher rotation speeds. This rather abrupt transition from a forced vortex motion to a free vortex motion occurs at Ro $\approx$ 0.6-0.8, consistent with Dunlap's data, and with HBG's data if the integral scale is viewed as the cylinder diameter. Similar vortices were seen in vortex tubes.[221]

It is important to note that right in the range of the transitional Rossby number, the PAM vehicles show rapid coning growth rate, as shown in Figure 21.

3.3 Nutational helical waves on vortices

The vortex after transition supports nutating, helical waves. While Dunlap has observed the spiral feature, the nutation of the helical waves on a vortex was reported by HBG, and later in greater detail by Maxworthy, Hopfinger, and Redekopp (referred as MHR hereafter). By disturbing the vortex induced by a suction tube in a spinning water tank MHR have observed the helical waves travelling along the vortex, as shown in Figure 22. As pointed out in the figure, as the helical wave travels along the vortex, it precesses in the

retrograde direction with the nondimensional precession frequency close to that of the PAM-D vehicle.

3.4 A Hypothesis for the PAM coning

Together, the above experimental results suggest a new hypothesis for the PAM coning, shown schematically in Figure 23. In this hypothesis, a rather abrupt transition from a forced vortex motion to a free vortex motion occurs inside the spinning solid rocket motor at a transitional Rossby number around 0.6-0.8. The vortex after transition supports helical waves, which, while traveling along the vortex, precess in the retrograde direction at a frequency close to that of the external coning excitation. This leads to a resonance between the nutational helical waves and the vehicle coning excitations and the subsequent rapid coning growth.

Unlike the jet gain model, the nutational helical disturbances in this hypothesis are not the perturbations of the solid body rotation. Consequently, no resonance is expected before the vortex transition. This is consistent with the flight data shown in Figure 3. However, despite the difference, the jet gain model by Flandro has stimulated the present work.

3.5 Recirculation vortices

As the swirling flow inside the solid rocket motor flows through the nozzle contraction, it generates a recirculating toroidal vortex. Such vortices can support helical oscillations, as shown in Figure 24. The resultant asymmetric pressure distribution around the cylinder wall is possible to drive the nutation of the spinning cylinder. More details on the oscillation frequency and travelling direction of the waves, however, are needed to make further evaluation of their relevance to the PAM coning. Since the three-dimensional

secondary flows associated with the recirculating vortices are known to have interaction with the central vortex,[238-240] it is reasonable to believe that the toroidal vortex may have some effect on the coning.

4. Concluding remarks

A new hypothesis has been developed for the PAM coning anomaly. In this hypothesis, a rather abrupt transition from a forced vortex motion to a free vortex motion occurs in the spinning rocket motor as the Rossby number decreases below a critical value around 0.6-0.8. The vortex after transition supports helical waves, which precess in the retrograde direction at a frequency close to the coning frequency of the PAM vehicle. This leads to a resonance between these two frequencies and the subsequent coning growth of the PAM vehicle. This hypothesis, based mainly on the experimental results, is consistent with the flight test data. This warrants further investigation of this hypothesis and its relationship with other models.

Helical gas flows have also been observed in the nozzle convergent section of the swirling supersonic jet, [261, 304] as shown in Figure 25. While no definite results are yet available to evaluate their relevance to the coning, the observation does support the request to extend current coning computations beyond the nozzle entrance of the solid rocket motor.[10]

So far in this review, only single phase flows have been examined. Since in the solid rocket motor the impingement of the oxidized metal particles on the chamber and nozzle entrance could account for 2 to 3 percent of the thrust loss,[305] it may not be negligible in the final coning calculation.

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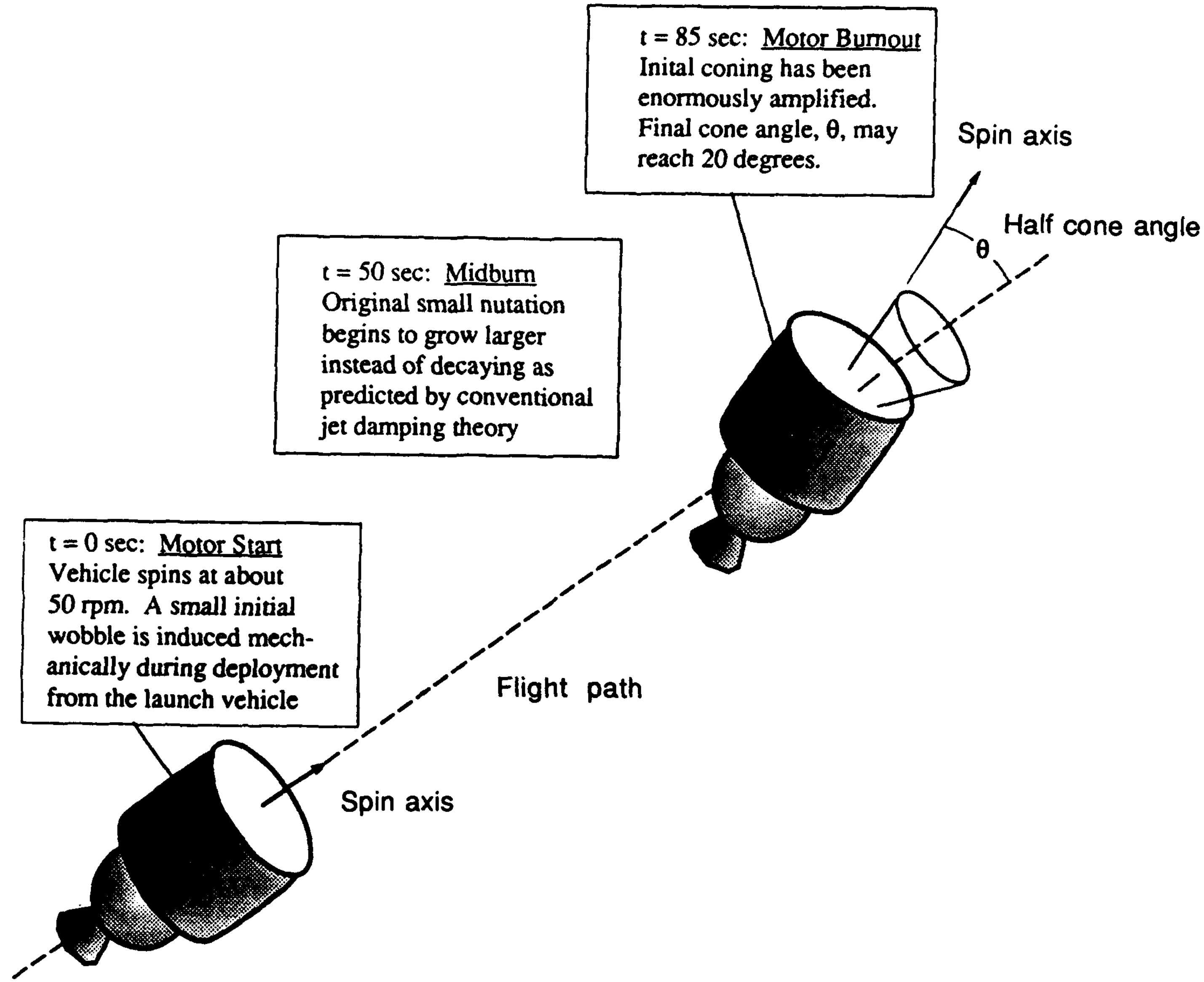


Figure 1 **Development of coning instability during motor burn of the STAR 48 motor of the PAM-D spacecraft. (Reference 8.)**

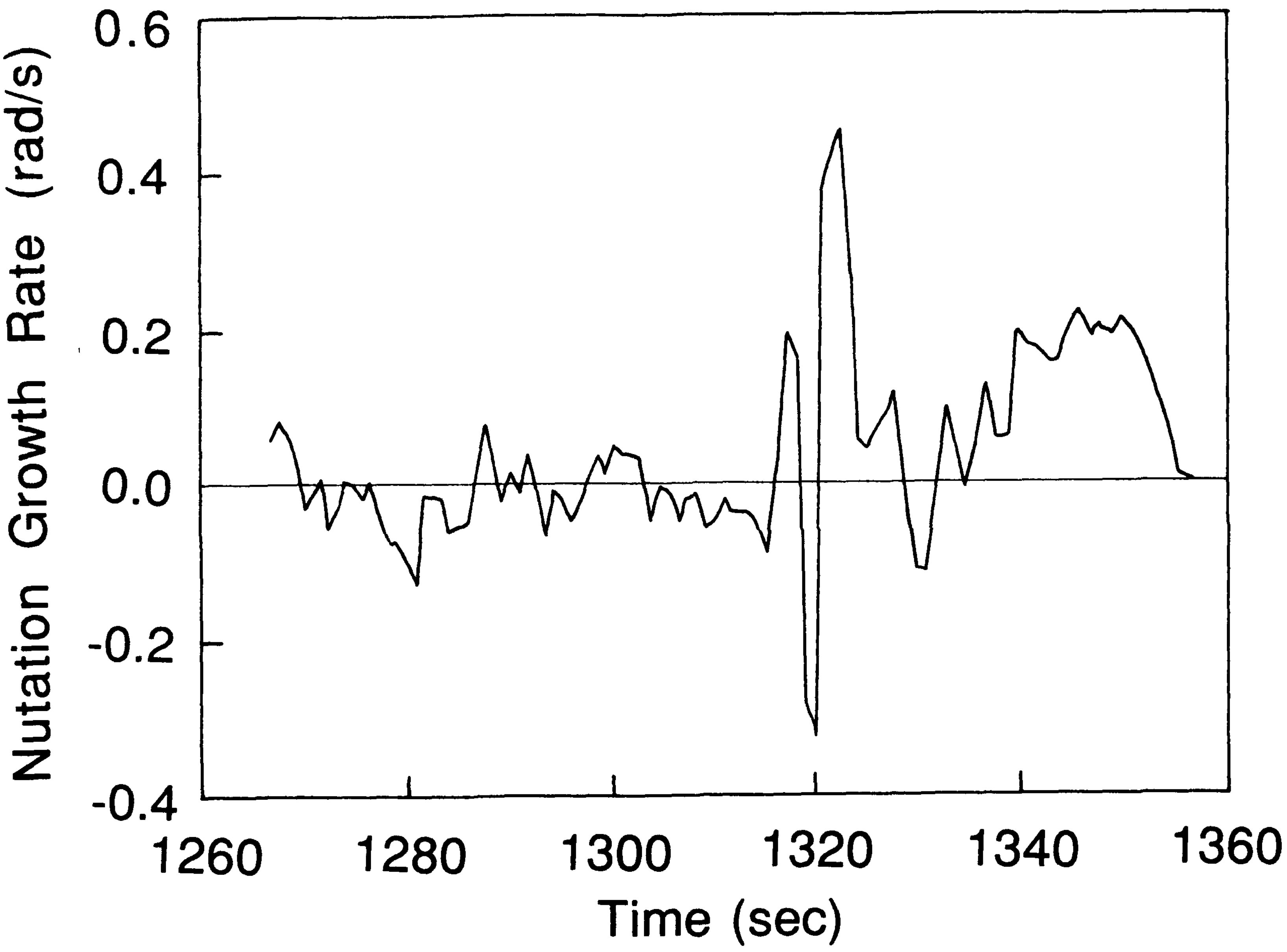


Figure 2 Typical time history of nutational growth rate of the WESTAR V spacecraft. (Reference 8.)

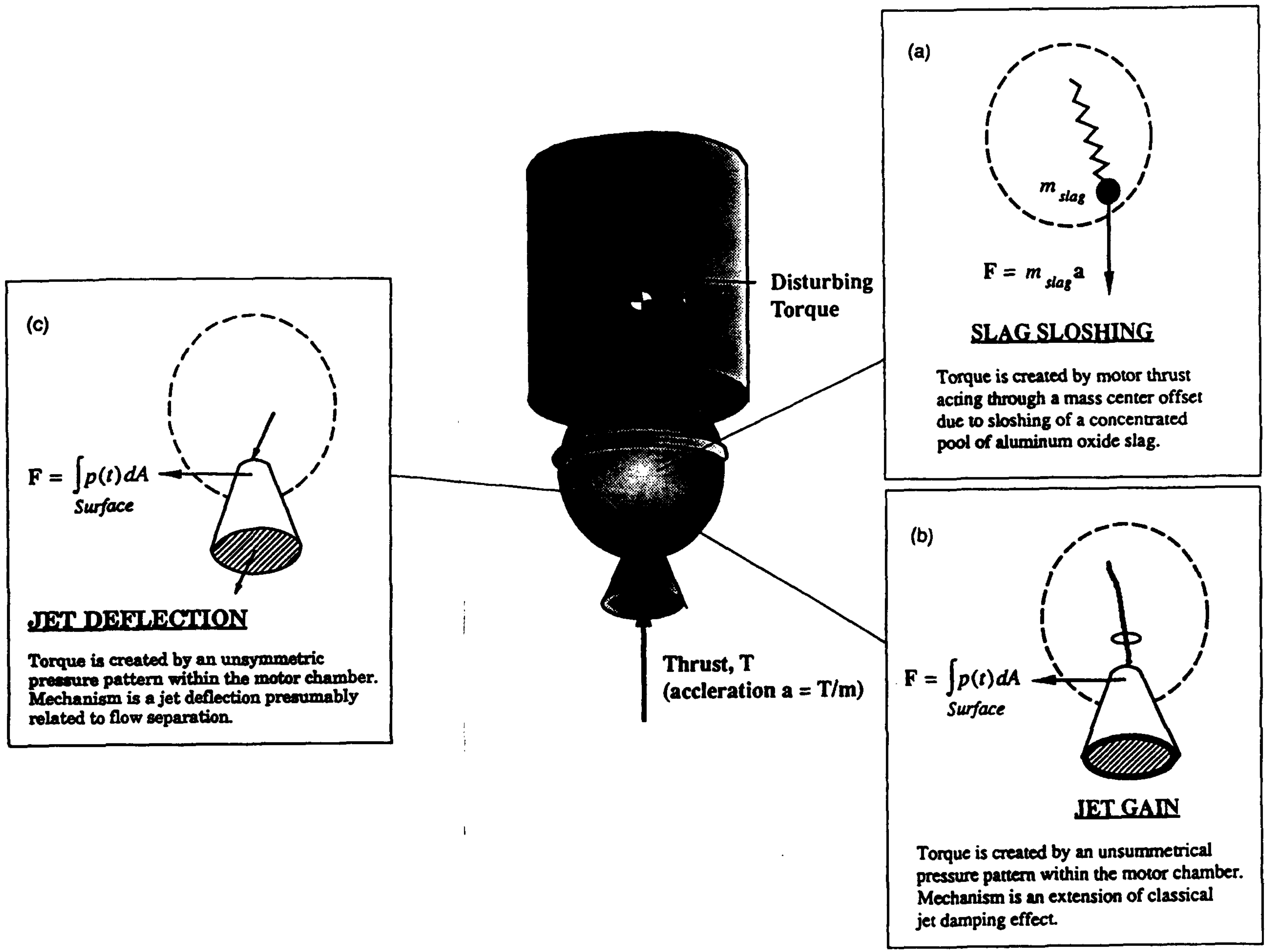
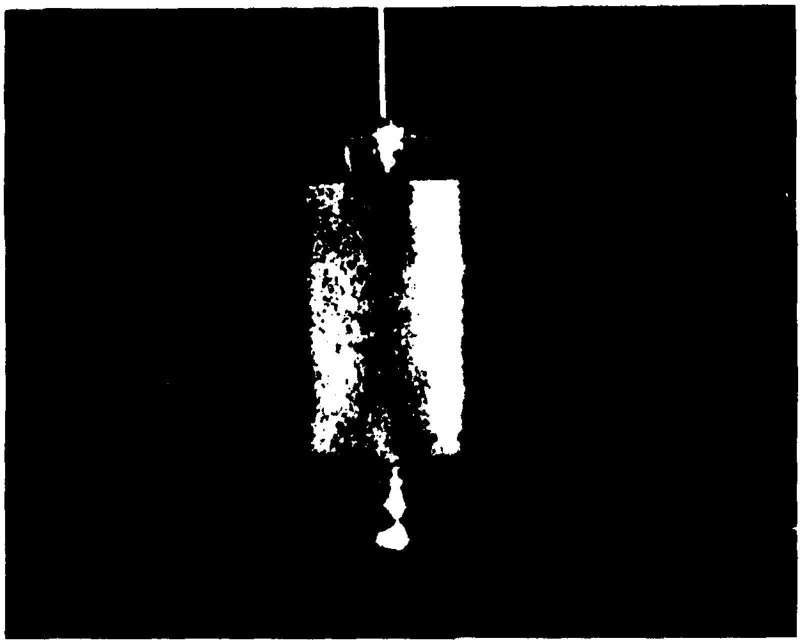
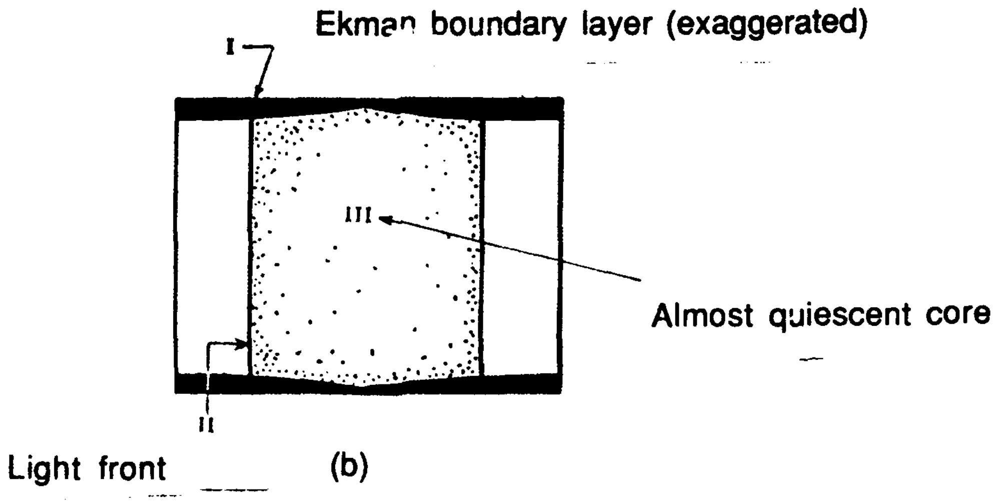


Figure 3 Proposed nutation mechanisms: (a) slag sloshing; (b) jet gain; and (c) jet deflection. (Reference 8.)



(a)



(b)

Figure 4 Spin-up from rest: (a) flow visualization; and (b) schematics of flow regimes. (Reference 18.)

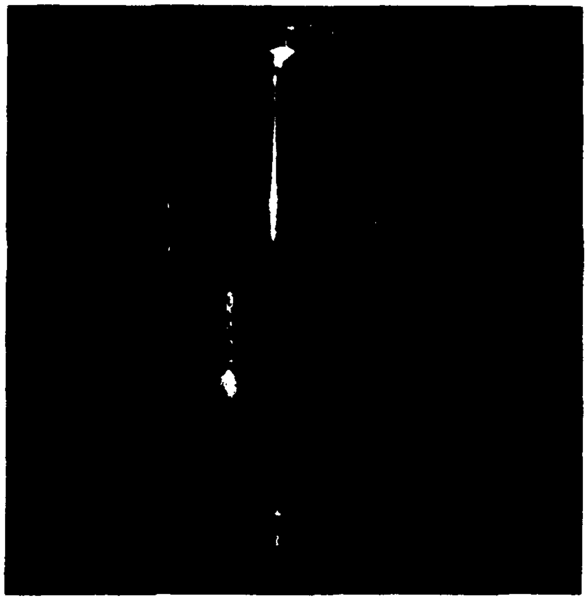


Figure 5 Axisymmetric waves produced by oscillating disk at $m/\Omega = 1.75$. The half cone angle is 59°. (Reference 30.)

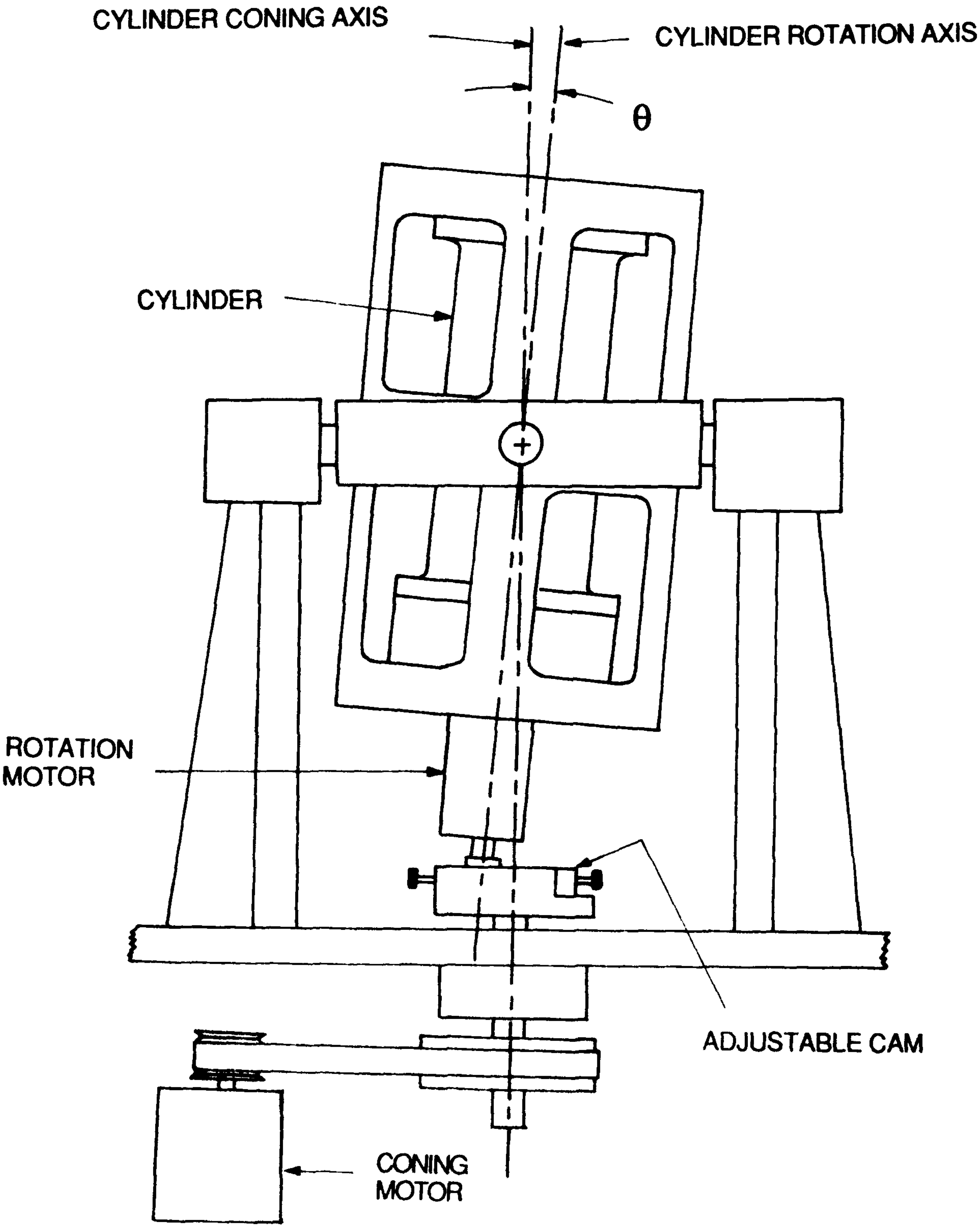


Figure 6 Schematics of a nutating, spinning cylinder. (Reference 104.)

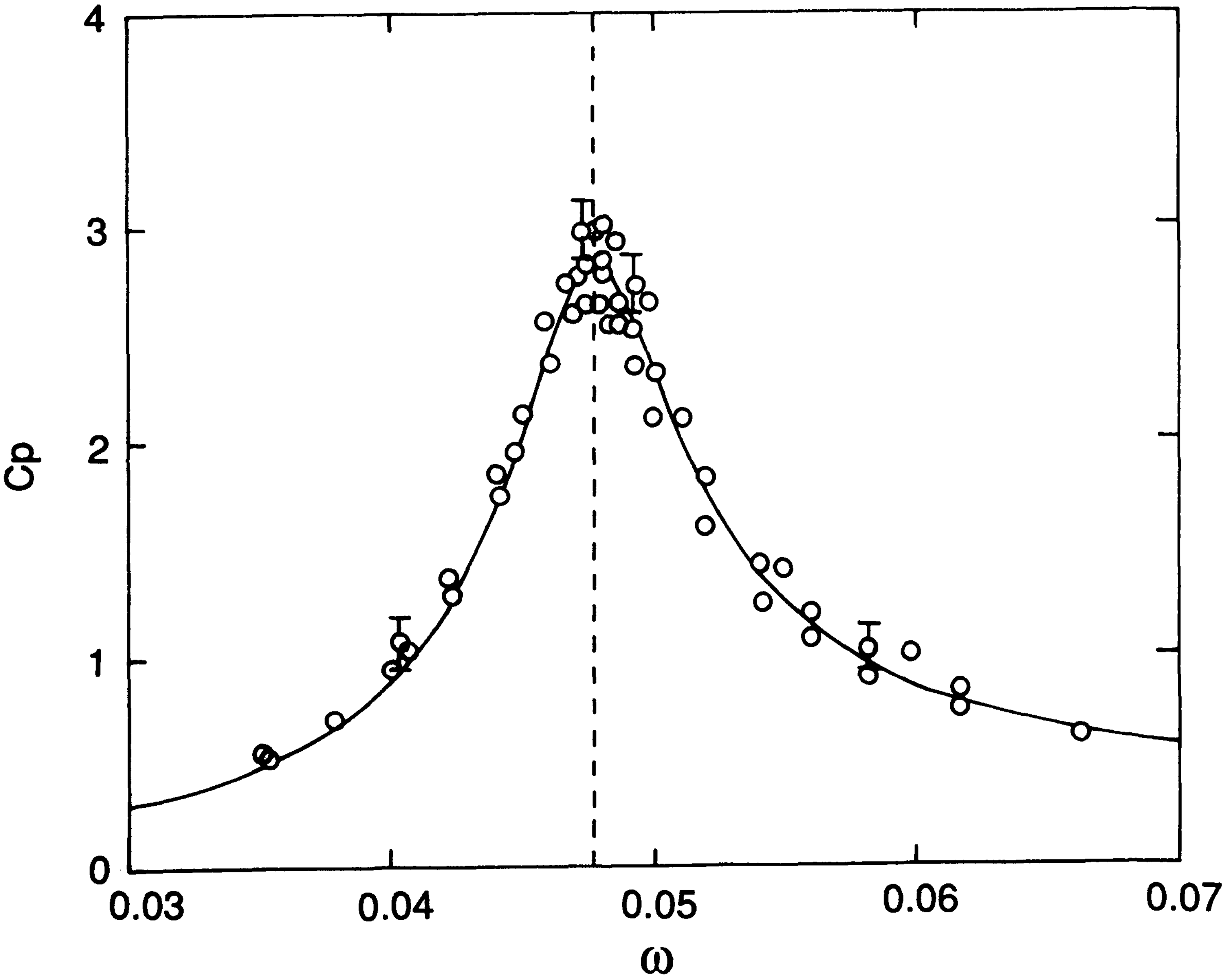
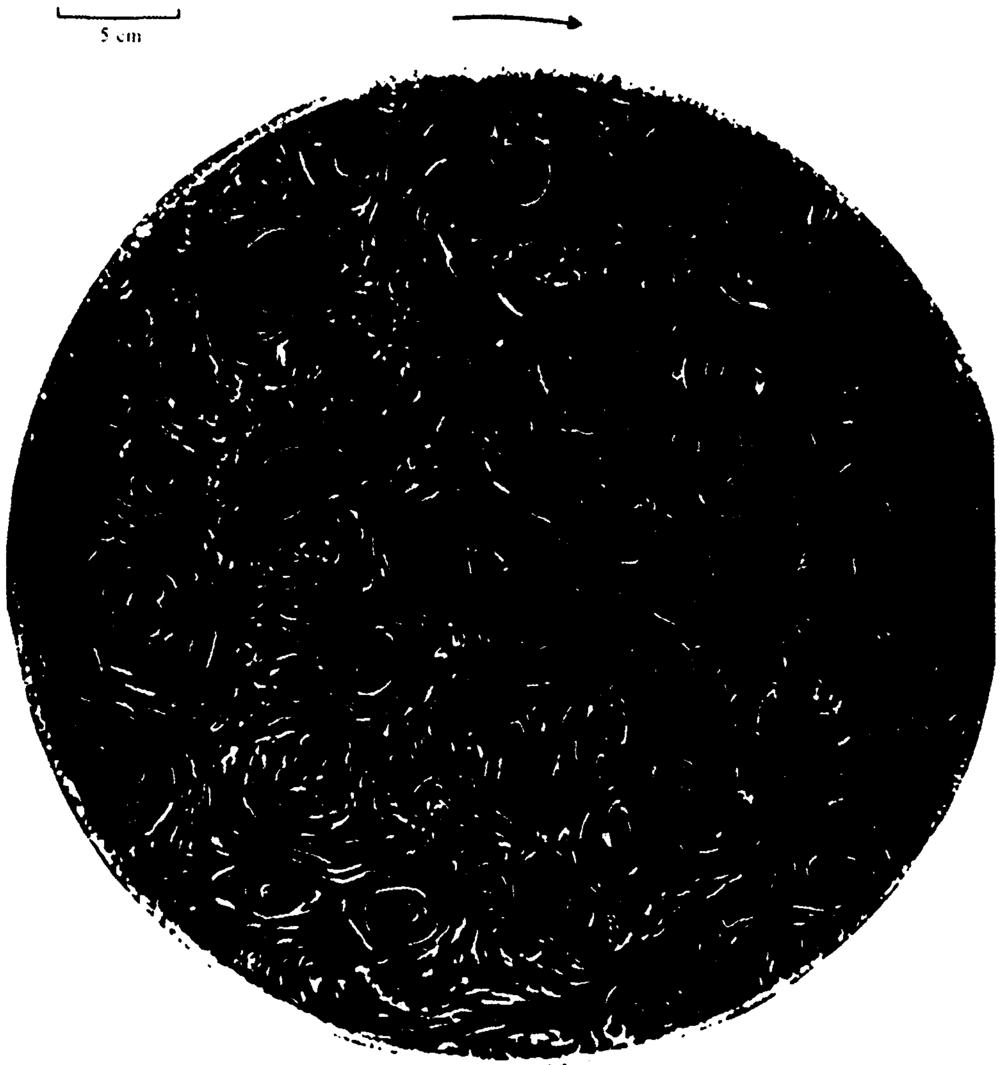


Figure 7 Cp vs ω. L/D = 3.15 and Re = 10^5. (Reference 104.)

Figure 8 Streakline photographs of the turbulent flow in a cross-section 30 cm above the grid midplane: (a) without tank rotation; and (b) with tank rotation. $\Omega = 2\pi$ rad s^{-1} and $n = 13.3\ \pi$ rad s^{-1}. (Reference 171.)



(b)

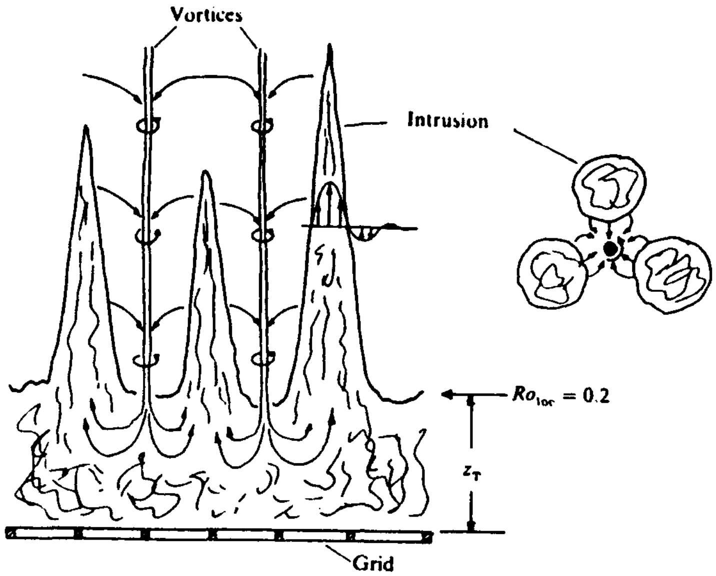


Figure 9 Formation of concentrated vortices by finger-like turbulence front. (Reference 173.)

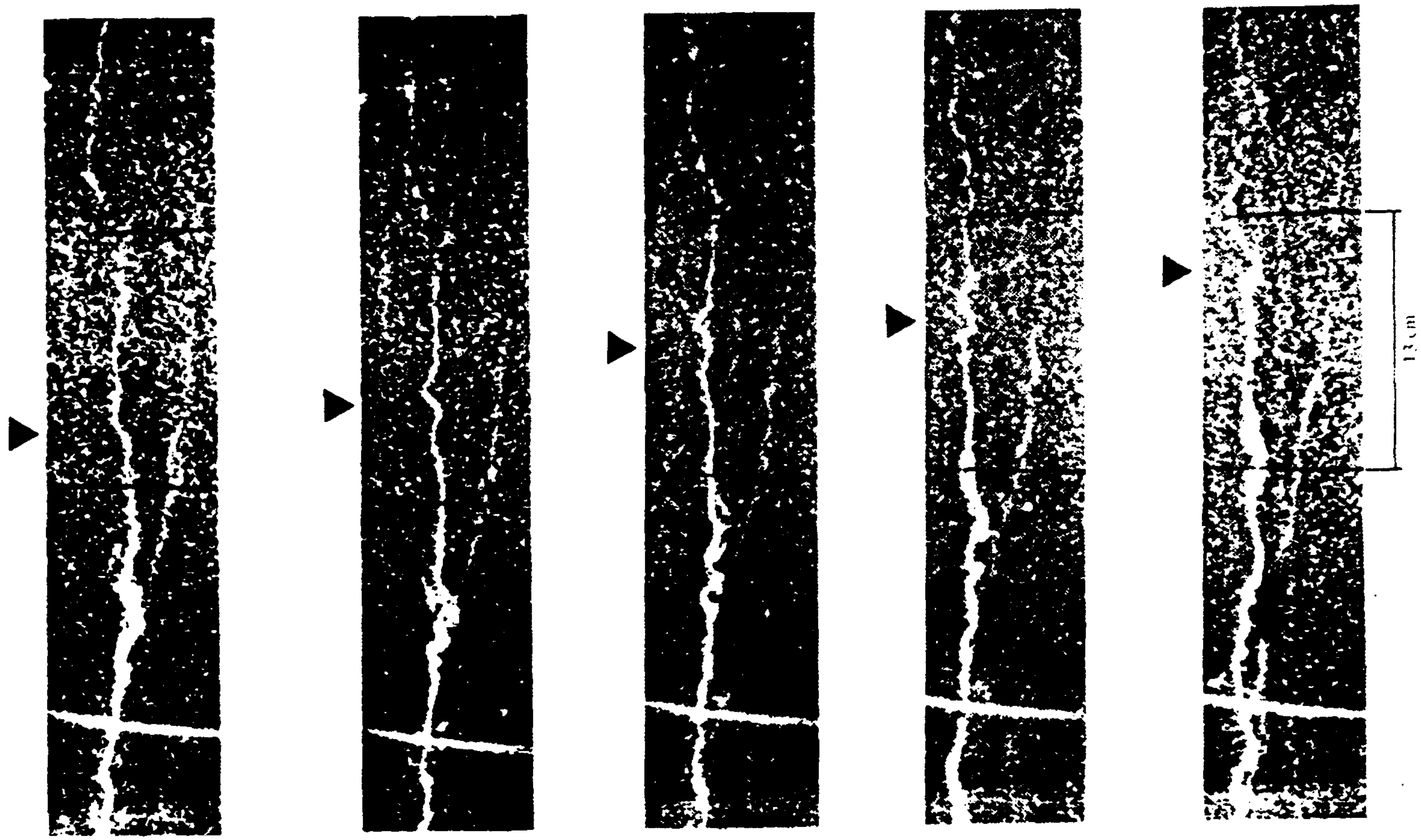


Figure 10 Time sequence of a helical wave travelling along a vortex. The mean position of the wave is marked by arrows. (Reference 171.)

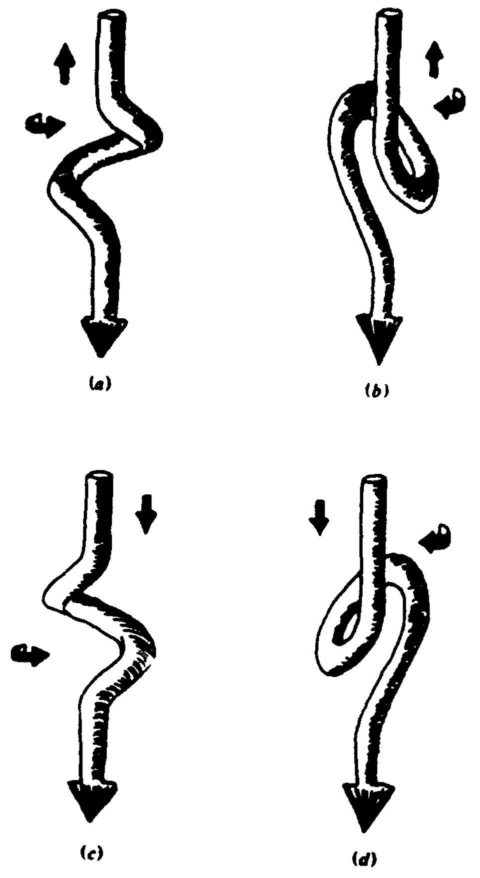


Figure 11. Schematics of possible wave shapes. Arrows indicate the direction of propagation/rotation of the wave patterns. (Reference ⁚71.)

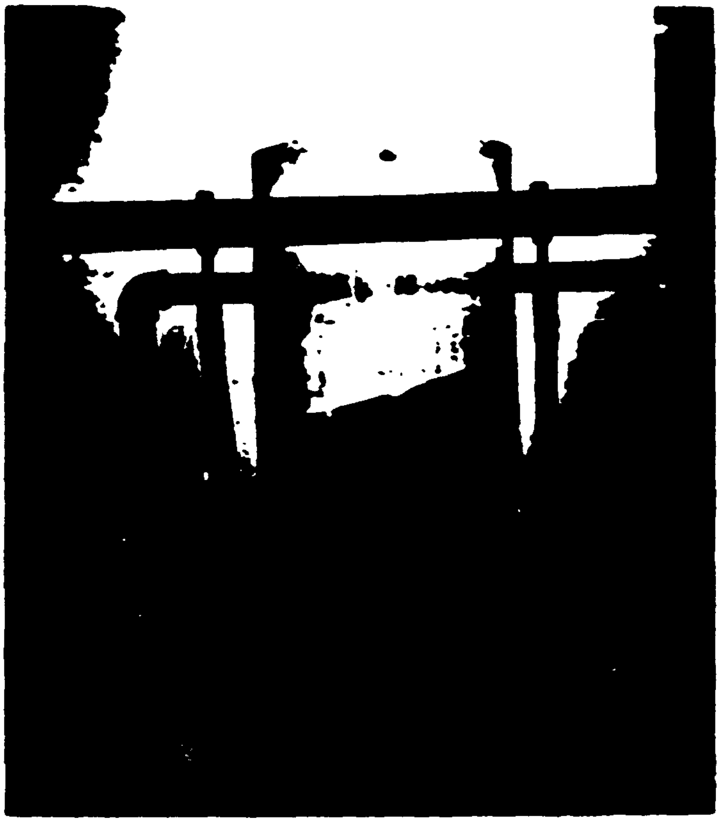


Figure 12. Liquid sloshing in an upright circular cylindrical tank. (Reference 195.)

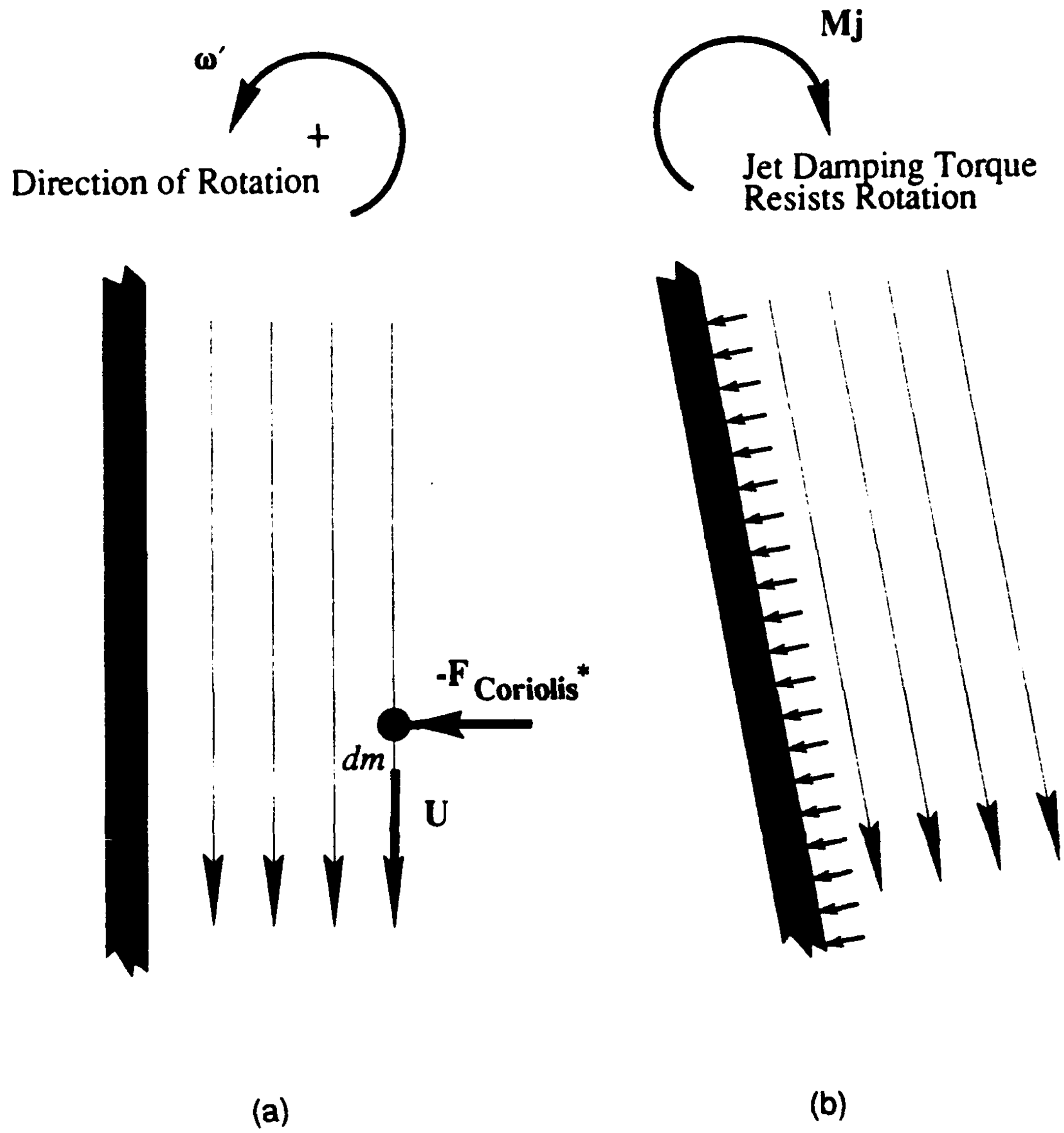


Figure 13 Classical jet damping effect: (a) uniform gas stream as chamber begins to rotate; (b) chamber in rotation; pressure forces are produced on wall to retain uniform gas flow. (Reference 8.)

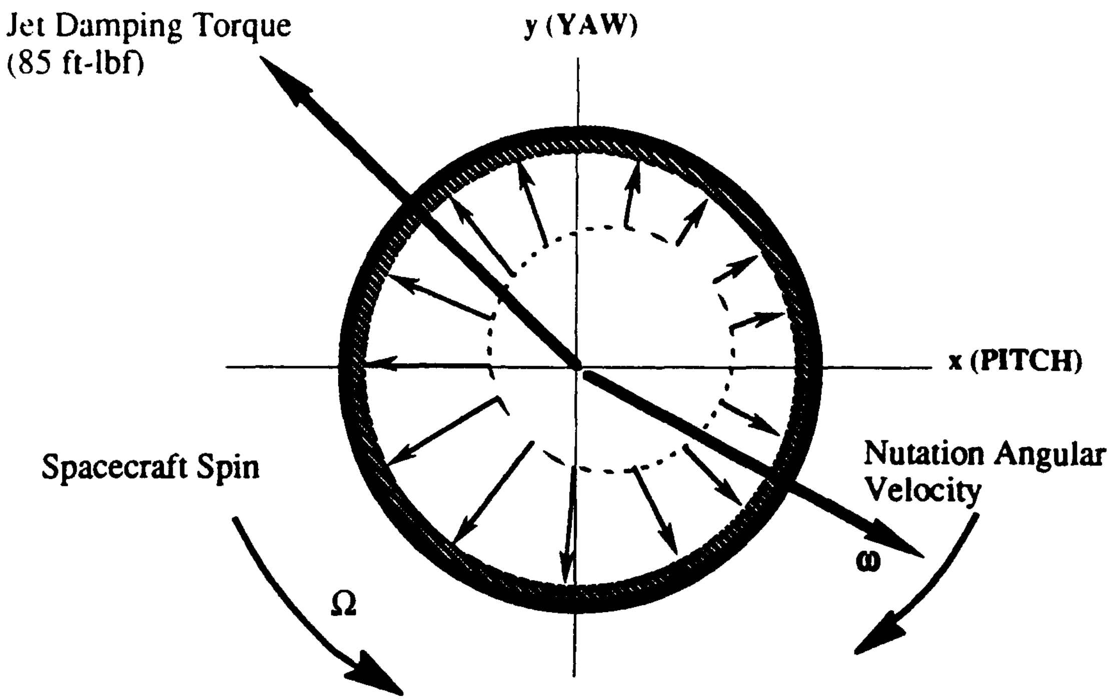


Figure 14 Reaction torque predicted by the jet damping theory. (Reference 8.)

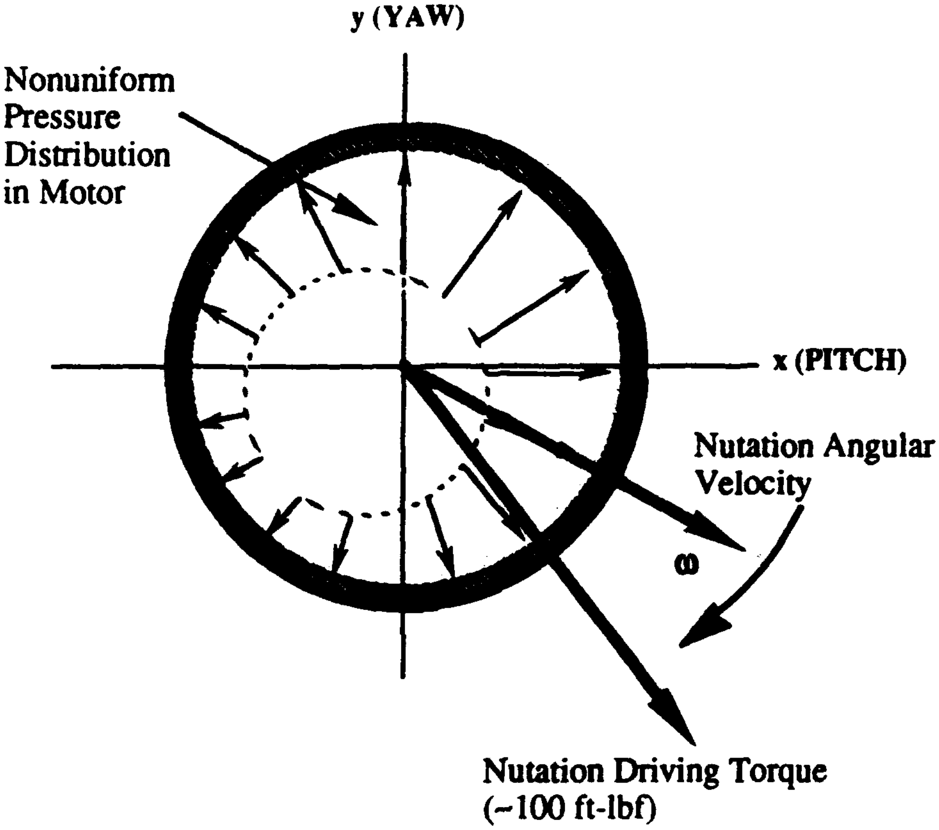


Figure 15 Driving torque in a nutating cylinder. (Reference 8.)

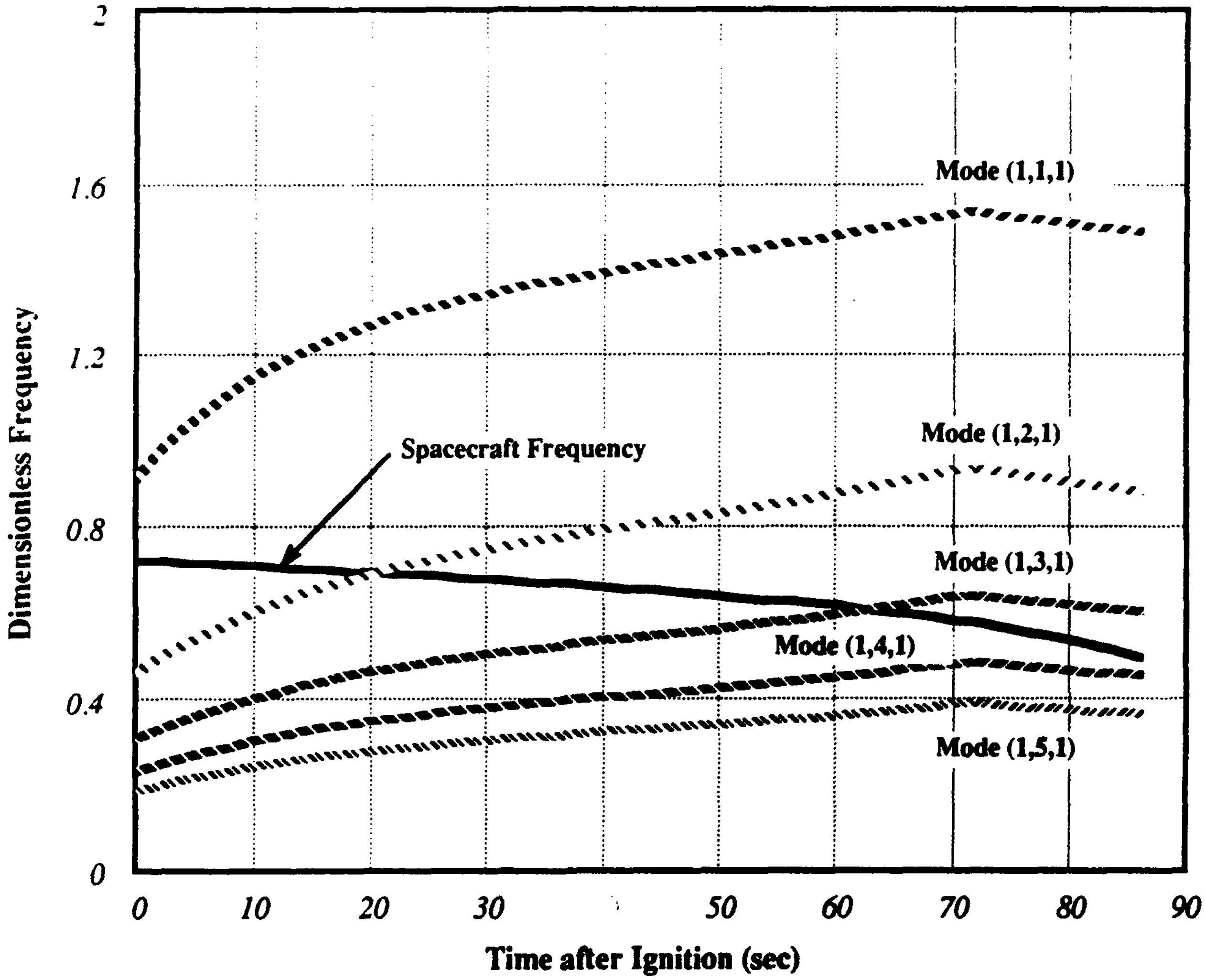


Figure 16 Two resonances predicted by the jet gain model. (Reference 8.)

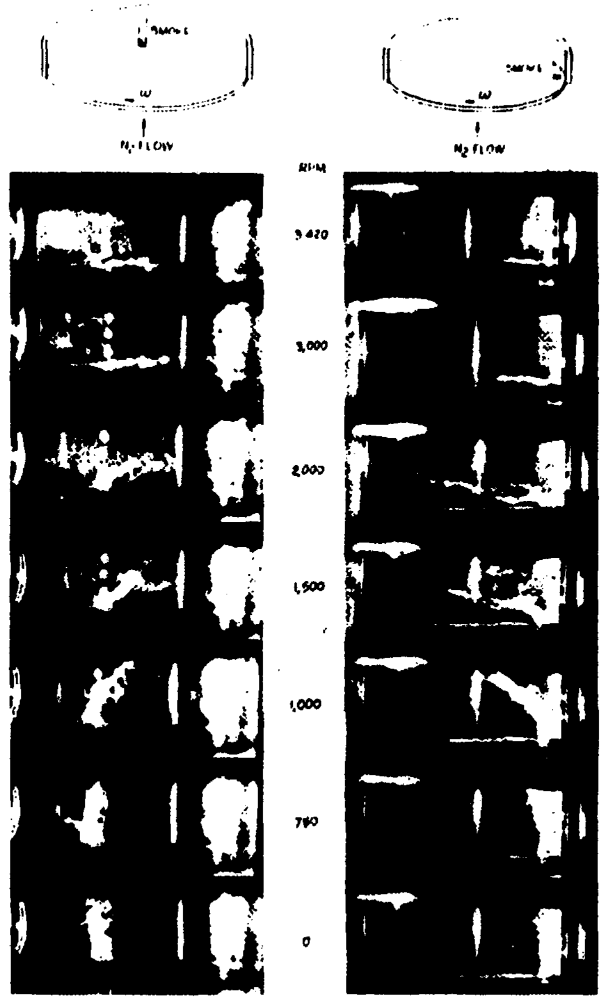


Figure 17 Photographs of smoke tracer in simulated spinning end-burner. Smoke port is 1.45 in from the centerline. (Reference 218.)

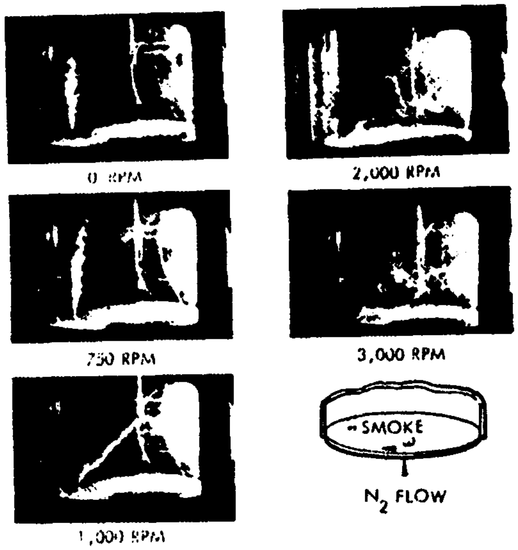


Figure 18 Photographs of smoke tracer in simulated spinning end-burner. Smoke port is 1.15 in from the centerline. (Reference 218.)

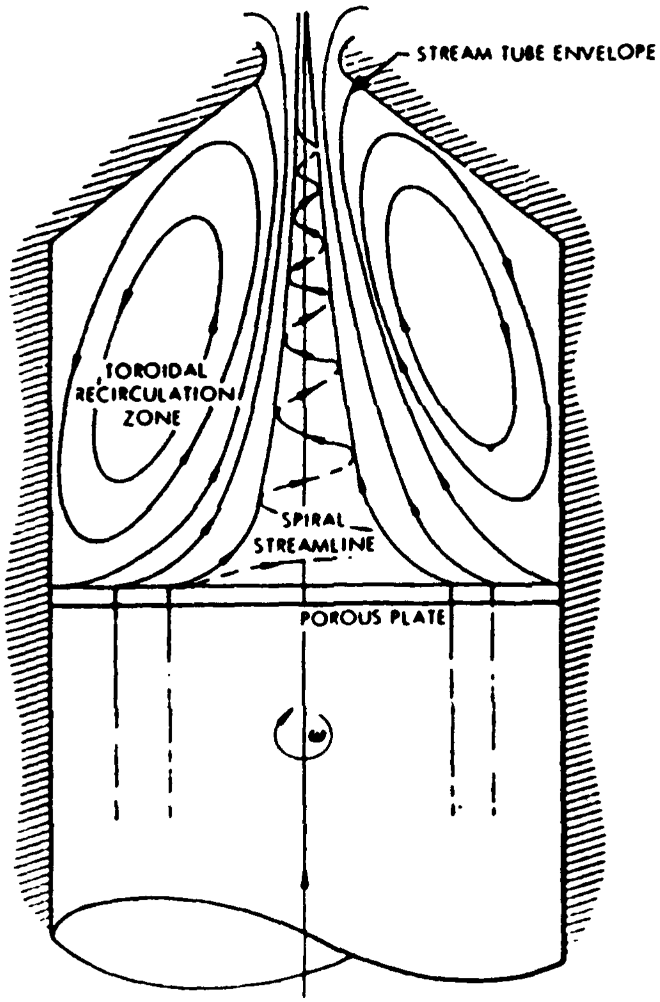


Figure 19 Spiral vortex flow at high motor spin rates. (Reference 218.)

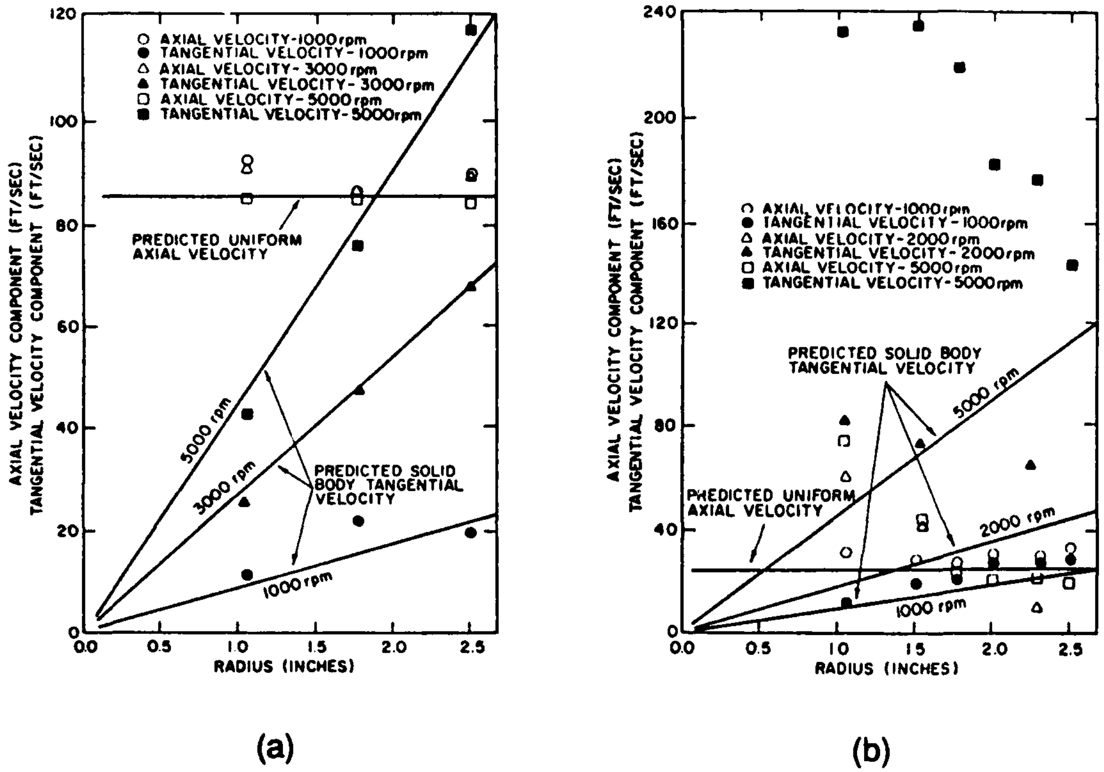


Figure 20 Velocity profiles in a cold-flow model of spinning solid rocket motor at contraction ratio: (a) 5.25; and (b) 22.1. (Reference 219.)

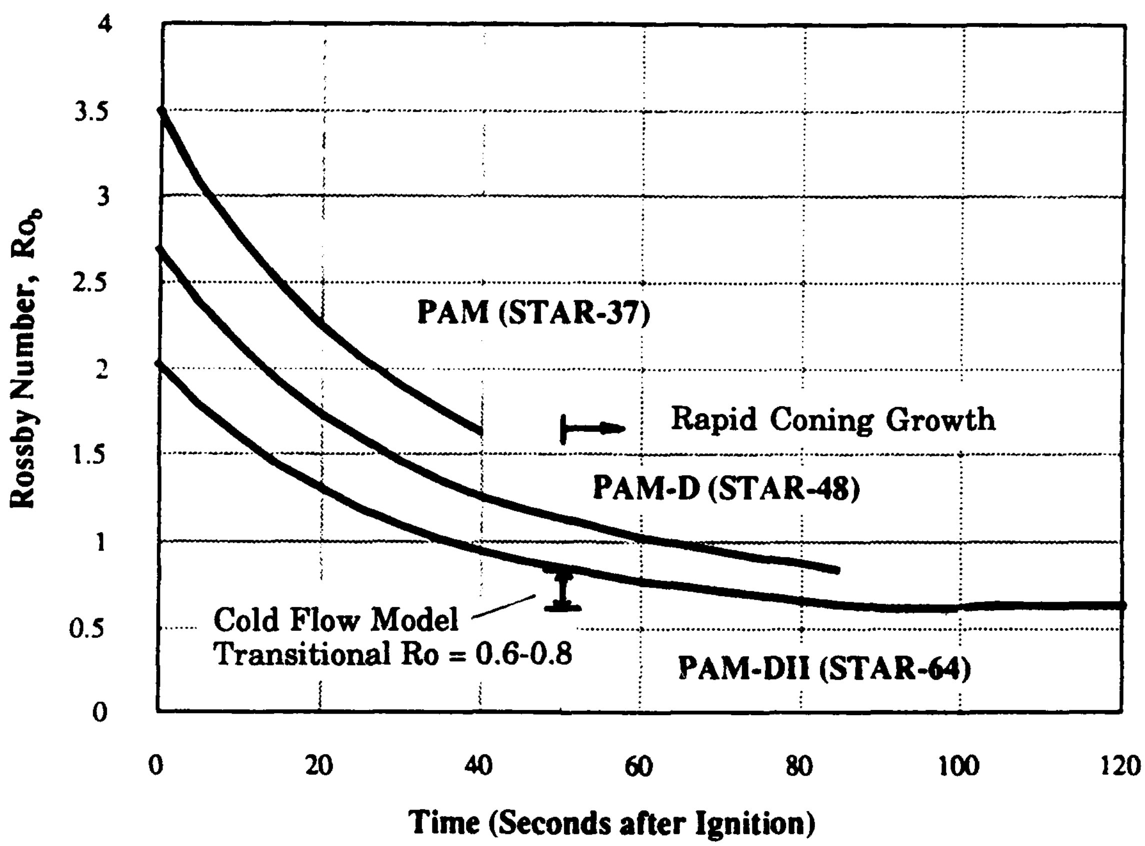
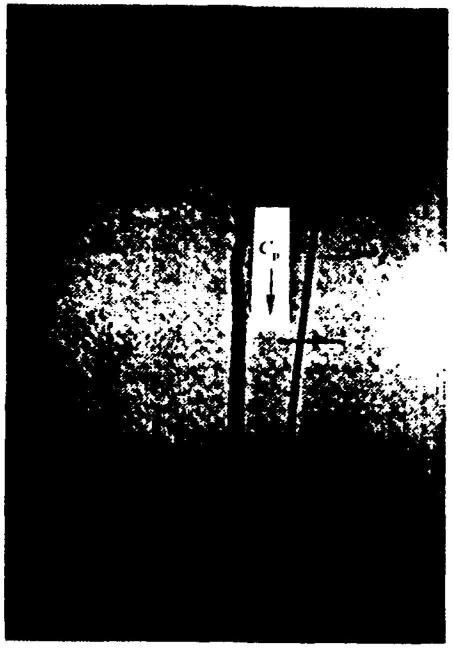


Figure 21 Time history of Rossby numbers of spinning solid rocket motors. (Reference 8.)



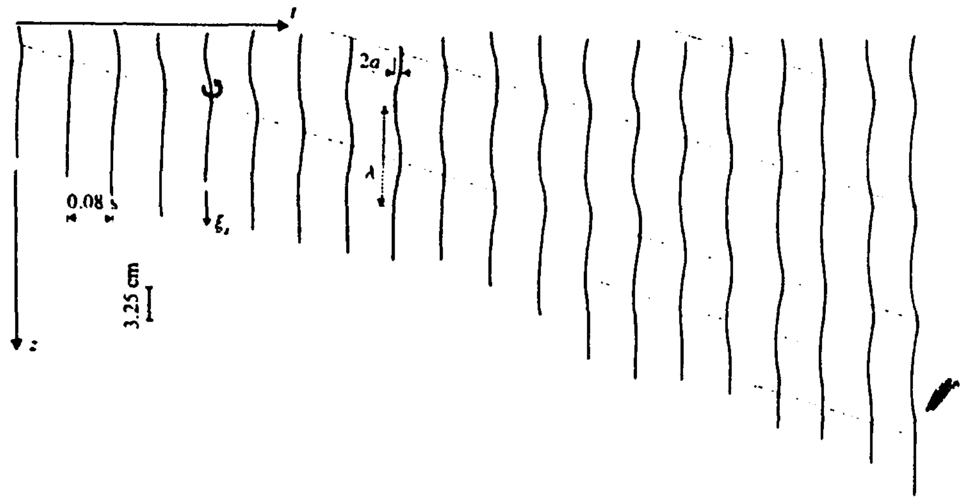
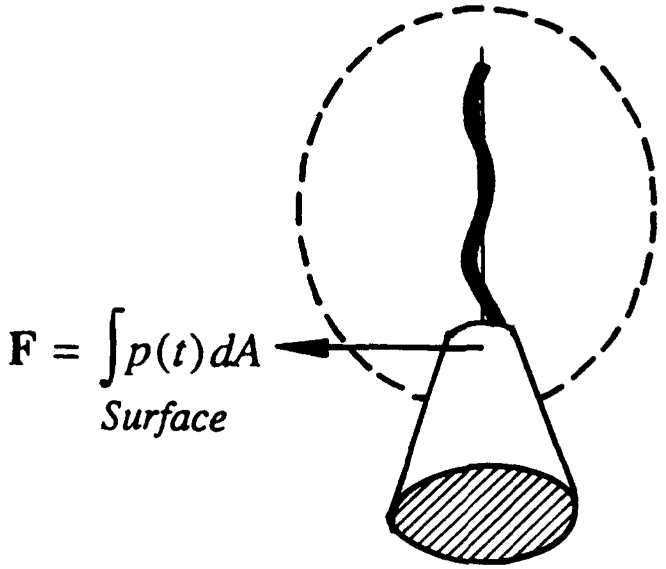


Figure 22. Photograph of a helica vortex filament wave, with time sequence of tracings of vortex centerlines. The rotation of the vortex filament indicated by ⟳ is opposite to the sense of vorticity of the undisturbed vortex. (Reference 173.)



NUTATING HELICAL VORTEX

Torque is created by an unsymmetric pressure pattern associated with nutating, helical vortex within the motor chamber.

Figure 23. Nutational helical vortex model for the PAM coning.

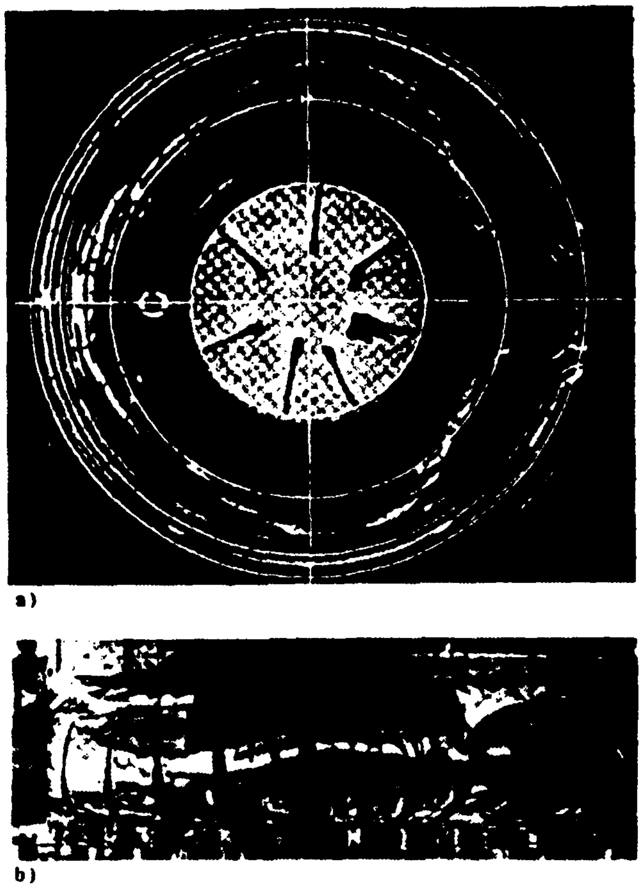


Figure 24. Asymmetric waves on confined toroidal vortex. (Reference 301.)

Figure 25 Swirl pattern in nozzle convergent section. (Reference 261.)